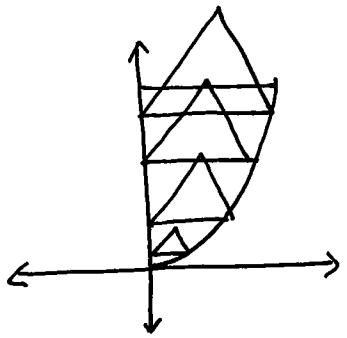
AP Calc AB
Volume Test Review

① 

$y = x^2$

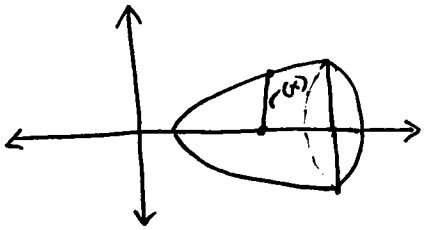
$x = \sqrt{y}$

Base: $\sqrt{y}$

$$\frac{\sqrt{3}}{4}\int_0^9 (\sqrt{y})^2\,dy$$

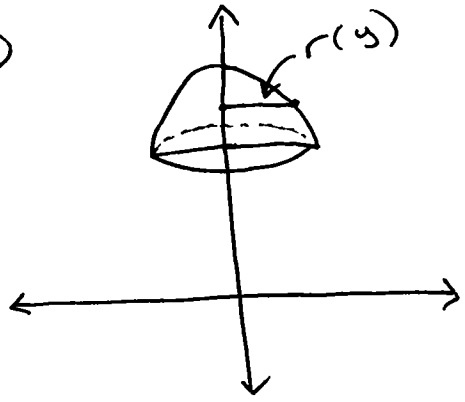
$$\frac{\sqrt{3}}{4}(40.5) =$$

17.537

② 

$r(x) = \ln(x)$

$$\pi\int_1^5 [\ln(x)]^2\,dx = \boxed{4.857\pi}$$

③ 

$r(y) = \sqrt{9-y}$

$$\pi\int_5^9 \left(\sqrt{9-y}\right)^2 dy$$

$y = 9 - x^2$

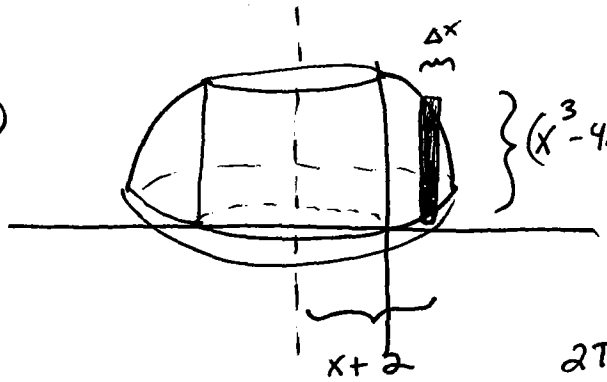
$$\pi\int_5^9 (9-y)\,dy = \pi\left[9y - \tfrac{1}{2}y^2\Big|_5^9\right]$$

$x = \sqrt{9-y}$

$$\pi\left[9(9) - \tfrac{1}{2}(9)^2 - \left(9(5) - \tfrac{1}{2}(5)^2\right)\right]$$

8π

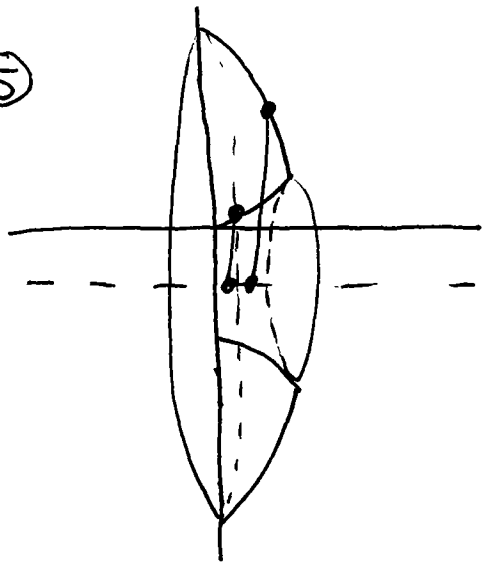
④



$$2\pi \int_0^1 (x+2)(x^3-5x^2+4)\,dx$$

$$\boxed{12.233\pi}$$

⑤



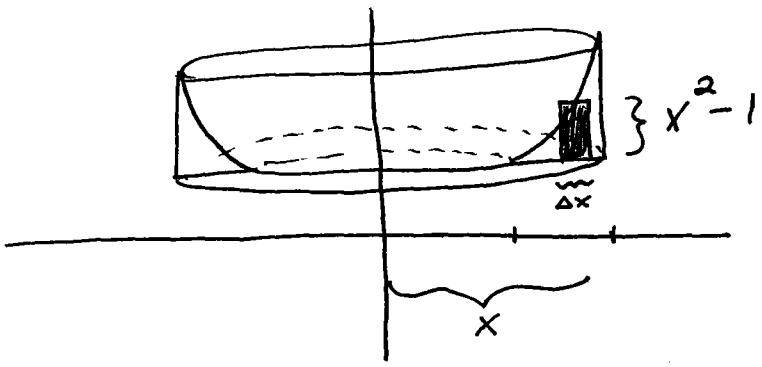
$$r(x) = x^2+1$$

$$R(x) = x^3-4x^2+4+1$$
$$= x^3-4x^2+5$$

$$\pi \int_0^1 \left[(x^3-4x^2+5)^2 - (x^2+1)^2\right] dx$$

$$\boxed{14.3095\pi}$$

⑥



Bounds:

$y = x^2$

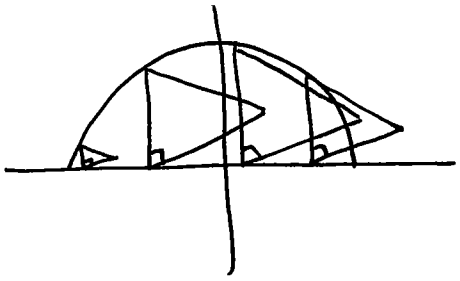
$1 = x^2$

$\boxed{x=1}$

$x=3$ ... Given

$$2\pi \int_1^3 x(x^2-1)\,dx = \boxed{32\pi}$$

(7)



Bounds: $\sqrt{16-x^2} = 0$

$x^2 = 16$

$x = \pm 4$

Base: $f(x) = \sqrt{16-x^2}$

$$\frac{1}{2}\int_{-4}^{4} \left(\sqrt{16-x^2}\right)^2 dx = \boxed{\frac{128}{3}}$$

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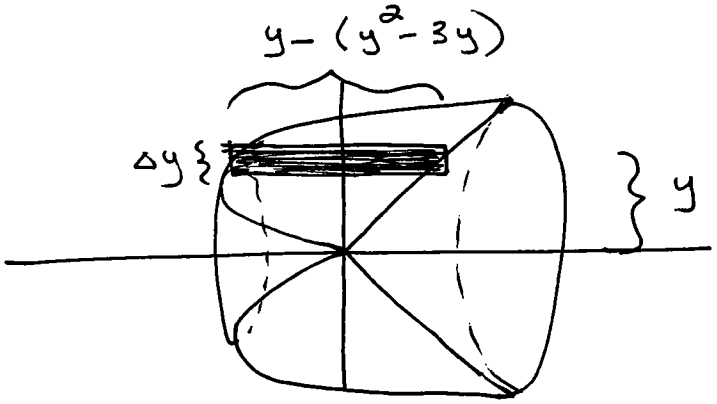
(8)



Bounds:

$y^2 - 3y = y$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0 \quad y = 4$

$$2\pi \int_{0}^{4} y\left(y-(y^2-3y)\right) dy = \boxed{\frac{128}{3}\pi}$$

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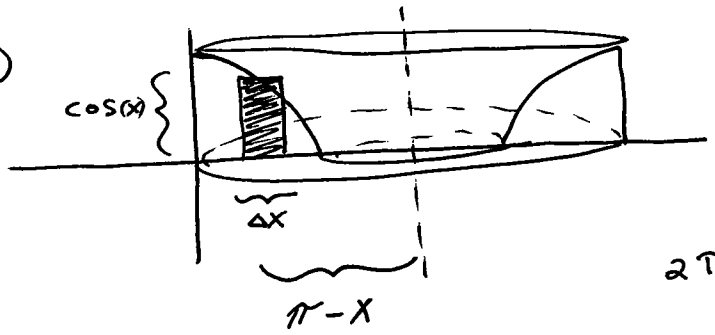
(9)



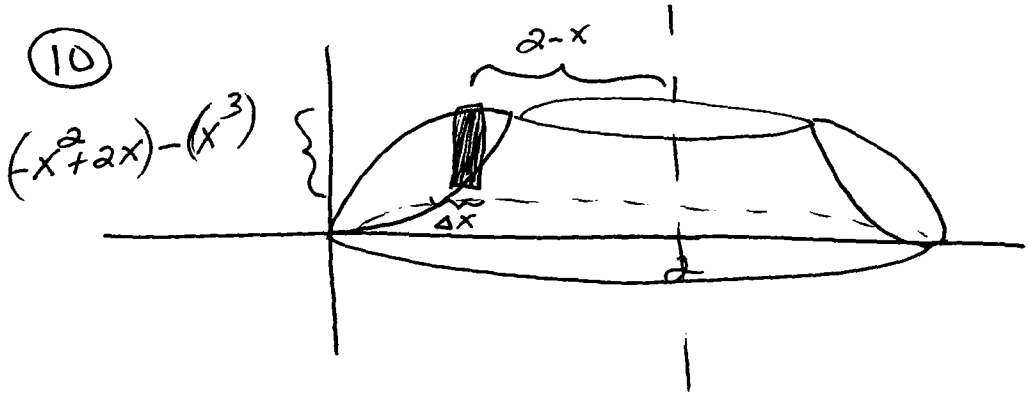
Bounds: $\cos(x) = 0$

$x = \frac{\pi}{2}$

$$2\pi \int_{0}^{\frac{\pi}{2}} \cos(x)(\pi - x)\, dx$$

$\boxed{5.1416\,\pi}$

(10)



Bounds: $-x^2+2x = x^3$

$x = -2, 0, 1$

not in domain

A) can't be done. Can't get x alone on 2nd function.

B) $$2\pi \int_0^1 (-x^3 - x^2 + 2x)(2-x)\,dx = \boxed{\frac{37}{30}\pi}$$

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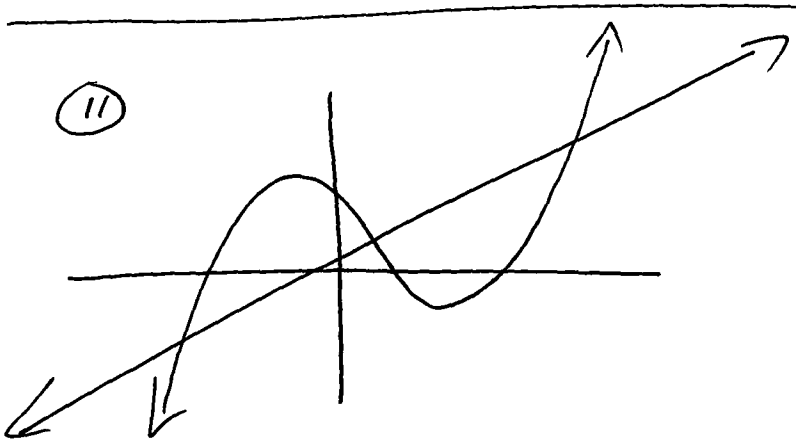
(11)



$g(x) = f(x)$

$x = -2.564$

$x = 0$

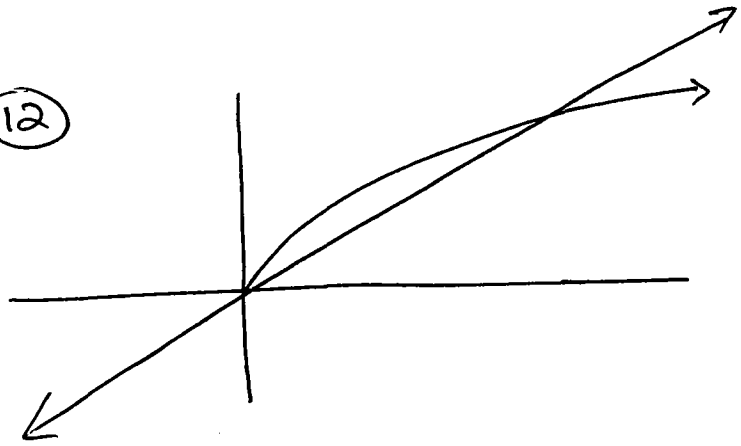
$x = 2.564$

$$\int_{-2.564}^{0} [f(x) - g(x)]\,dx + \int_{0}^{2.564} [g(x) - f(x)]\,dx$$

$8.529 + 8.529$

$$\boxed{17.058}$$

(12)



Bounds:

$f(x) = g(x)$

$x = 0, 16$

$$\int_0^{16} \left(2\sqrt{x} - \frac{1}{2}x\right) dx = \boxed{21.333}$$

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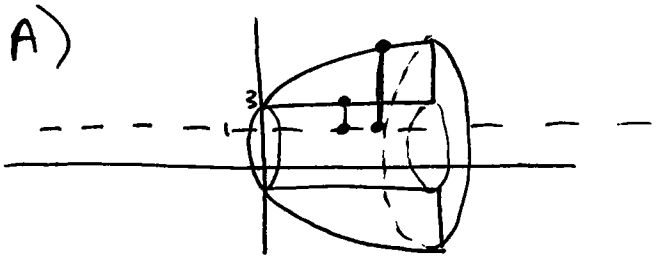
(13) A)



$r(x) = 2$

$R(x) = (\sqrt{x} + 3) - 1$

$\sqrt{x} + 2$

$$\pi \int_0^4 \left[(\sqrt{x}+2)^2 - 2^2\right] dx = \boxed{29.333\,\pi}$$

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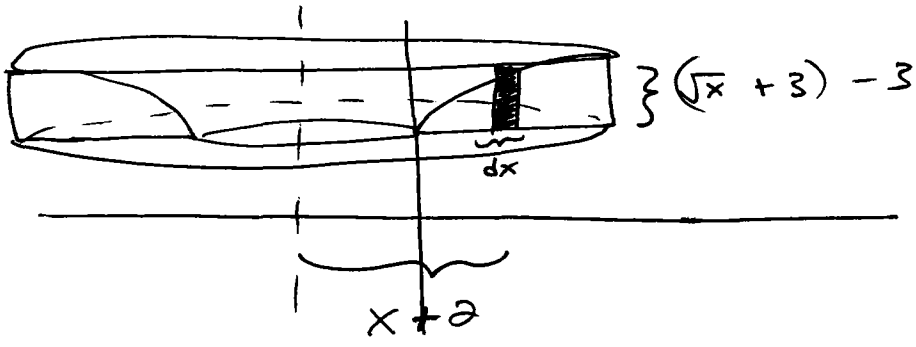
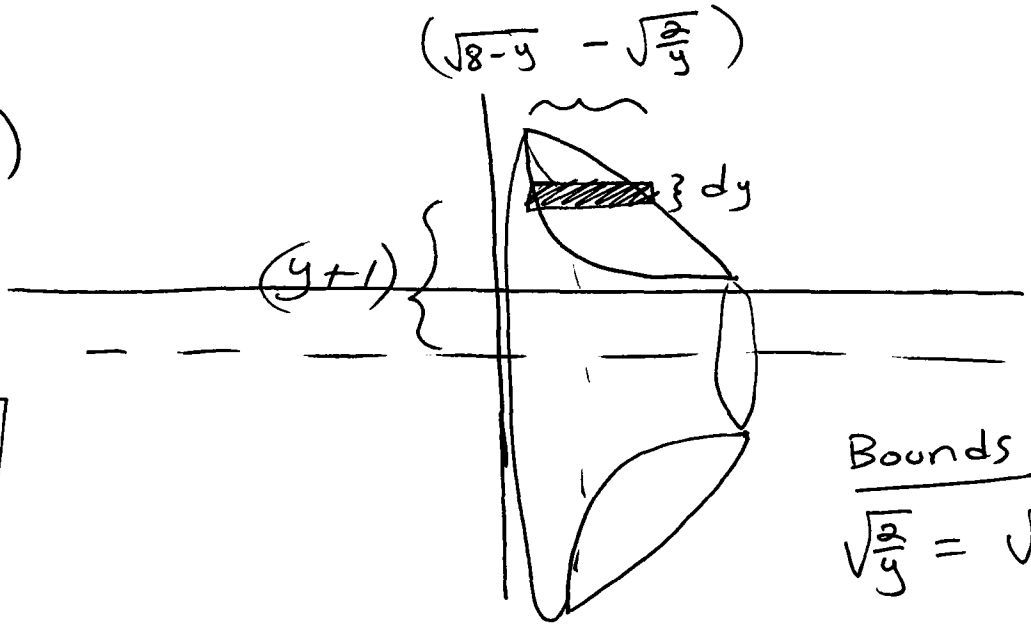
(13) B)



$\left.\begin{matrix}\\ \\ \end{matrix}\right\} (\sqrt{x} + 3) - 3 = \boxed{\sqrt{x}}$

$$2\pi \int_0^4 (x+2)(\sqrt{x})\, dx = \boxed{46.9332\,\pi}$$

(14) A)



$y = \frac{2}{x^2}$

$x = \sqrt{\frac{2}{y}}$

$y = -x^2 + 8$

$x = \sqrt{8-y}$

Bounds:

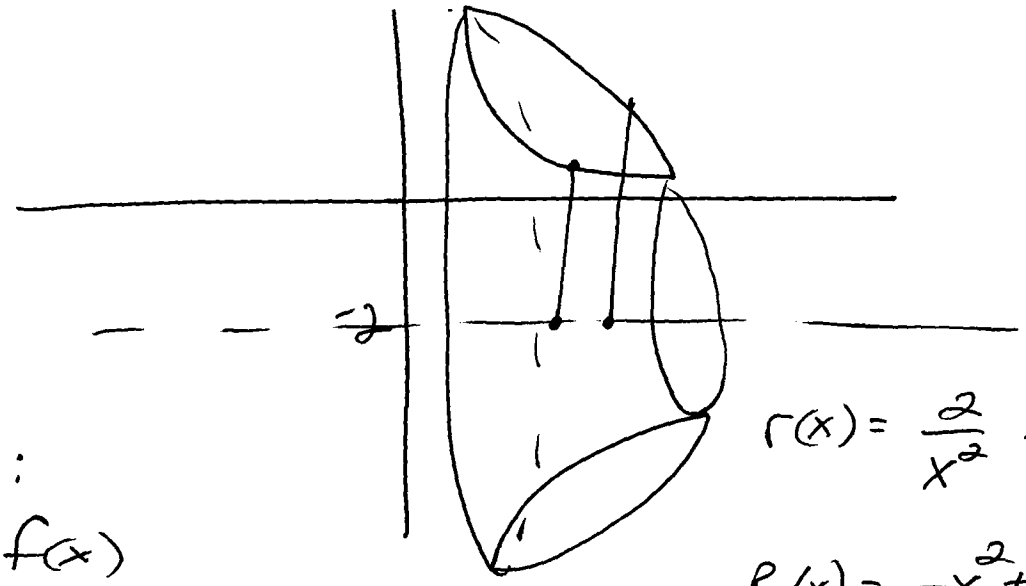
$\sqrt{\frac{2}{y}} = \sqrt{8-y}$

$y = .258$

$y = 7.742$

$$2\pi \int_{.258}^{7.742} (y+1)\left(\sqrt{8-y} - \sqrt{\frac{2}{y}}\right) dy = 70.368\pi$$

(14) B)



$r(x) = \frac{2}{x^2} + 2$

$R(x) = -x^2 + 8 + 2$

$= -x^2 + 10$

Bounds:

$g(x) = f(x)$

$x = .508$

$x = 2.782$

$$\pi \int_{.508}^{2.782} \left(-x^2+10\right)^2 - \left(\frac{2}{x^2}+2\right)^2 \, dx$$

$$85.976\pi$$